# Relativistic Transformation of Velocities and its Effects on Black Body radiation

Dhruva Sambrani

March 23, 2019

Let S and S' be two frames such that S' has a uniform velocity of magnitude $u\hat{\mathbf{i}}$ wrt S, and at $t = 0$, origin of S and S' coincide. Further, let $(x, y, z, t)$ *and* $(x', y', z', t')$ describe an event in S and S' respectively, and since frames are in uniform velocity with respect to each other, (S')=LC(S). $\gamma = \frac{1}{\sqrt{1-(\frac{v}{c})^2}}$

We know from Lorentz transformations:

$$x' = \gamma(x - ut)$$
$$y' = y$$
$$z' = z$$
$$t' = \gamma(t - \frac{ux}{c^2})$$

So differentiating $t'$ $wrt$ $t$:

$$\frac{dt'}{dt} = \gamma(1 - \frac{uv_x}{c^2})$$

Now differentiating $x',\ y',\ z',\ wrt\ t'$:

$$\frac{dx'}{dt'} = v'_x = \gamma(\frac{dx}{dt'} - u\frac{dt}{dt'})$$
$$\frac{dy'}{dt'} = v'_y = (\frac{dy}{dt'})$$
$$\frac{dz'}{dt'} = v'_z = (\frac{dz}{dt'})$$

Substituting $dt'$ with $dt$ in $RHS$:

$$v'_x = \frac{v_x - u}{1 - \frac{uv_x}{c^2}}$$
$$v'_y = \frac{v_y}{\gamma(1 - \frac{uv_x}{c^2})}$$
$$v'_z = \frac{v_z}{\gamma(1 - \frac{uv_x}{c^2})}$$

Hence we have a relativistic transformation of velocities.

Consider a particle with is emitting Black Body Radiation. Let S' be the particle (rest) frame. Also let S be the lab frame, in which particle moves with $u\hat{\mathbf{i}}$. Also let R be a ray of EM radiation, which has angle $\theta$ wrt x axis and $\theta'$ wrt x' axis. Since ray is EM:

$$v = c$$
$$v_x = c\ cos(\theta)$$
$$v_{yz} = c\ sin(\theta)$$
$$v'_x = c\ cos(\theta')$$
$$v'_{yz} = c\ sin(\theta')$$

Hence dividing $v_{yz}$ and $v_x$, and substituting them with $v'_{yz}$ *and* $v'_x$ respectively:

$$tan(\theta) = \frac{v'_{yz}}{\gamma(v'_x + u)}$$
$$\implies tan(\theta) = \frac{sin(\theta')}{\gamma(cos(\theta') + \frac{u}{c})}$$

Hence, we have seen how the direction of the EM Ray transforms relativistically. It must be noted that the speed of EM Ray is still a constant, $c$, in accordance to our initial assumptions. Now Plotting transformed rays gives us the following result-