Quantitative Analysis of the Force on a Black Body Moving in the Lab Frame

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Let S and S' be two frames such that S' has a uniform velocity of magnitude $u\hat{\mathbf{i}}$ wrt S, and at $t = 0$, origin of S and S' coincide. Further, let (x, y, z, t) *and* (x', y', z', t') describe an event in S and S' respectively, and since frames are in uniform velocity with respect to each other, (S')=LC(S). $\gamma = \frac{1}{\sqrt{1-(\frac{v}{c})^2}}$

We know[1] that the direction of the EM Ray transforms relativistically by the following formula -

$$tan(\theta) = \frac{sin(\theta')}{\gamma(cos(\theta')+\frac{u}{c})}$$

where a ray of EM radiation, has angle θ wrt x axis and θ' wrt x' axis.

Let us focus our attention on the radiation in S'. From Stephan-Boltzmann's law, we have Power Radiated

$$P = \sigma AT^4$$

We can assume that each photon emitted by the particle has equal wavelength, λ_{avg}. Thus each photon has Energy

$$E = \frac{hc}{\lambda_{avg}}$$

Hence, the number of photons emitted per unit time

$$\frac{dn}{dt} = \frac{P}{E} = \frac{\sigma AT^4\lambda_{avg}}{hc}$$

From symmetry, each point in space has equal number of photons. Hence the area density of the photons on a spherical surface of radius R is

$$\rho = \frac{\sigma AT^4\lambda_{avg}}{4\pi R^2 hc}$$

Now, consider a thin circular "strip" on the surface subtending an angle θ' at the origin and centered around the x' axis.The number of photons passing through this strip is

$$N = \rho\ da = \rho\ 2\pi R\sin(\theta')Rd\theta' = \frac{\sigma AT^4\lambda_{avg}}{2hc}\sin(\theta')\ d\theta'$$

[1]Refer pg. 2 Velocity Transformations, Dhruva Sambrani

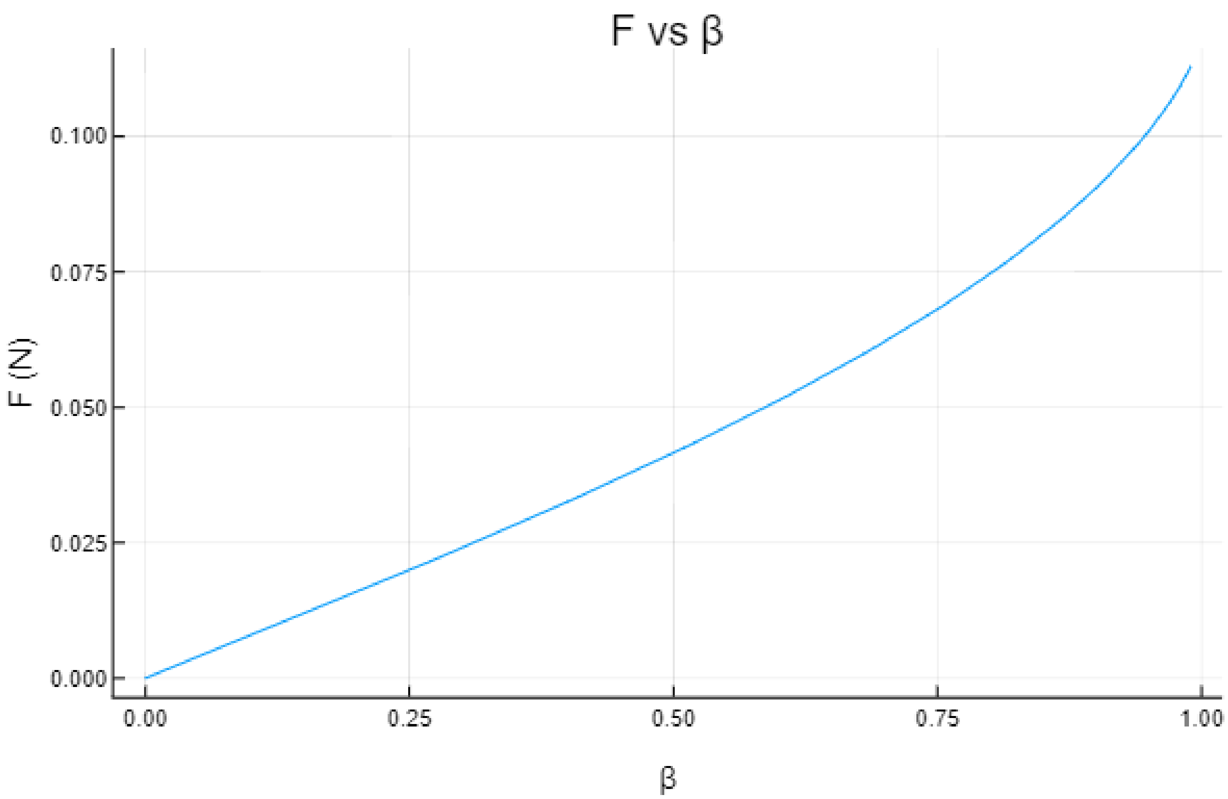
In S, all[2] these photons on the strip will now subtend an angle θ with the x axis. The sum of their momentum will be only along the x axis, as all perpendicular components cancel out by symmetry. The component of the momenta per unit time along the x axis is

$$dF = \frac{dp_x}{dt} = \frac{N\ h}{\lambda_{avg}} \cos(\theta) = \frac{\sigma A T^4}{2c} \sin(\theta') \cos(\theta)\ d\theta'$$

Hence the net force on the particle

$$F = \int dF = \frac{\sigma A T^4}{2c} \int_0^\pi \sin(\theta') \cos(\theta)\ d\theta'$$

We can now find the θ for every θ' by using the transformation function. This is a cumbersome integral, and it has been numerically solved. The result of the computation[3] has been shown in the following graph of Force in Newtons vs β -



[2]As the transformation function is an odd function in θ'

[3]Refer to Phy Winter Project.ipynb, Dhruva Sambrani, $T = 5000K,\ A = 1m^2$